Available online at: http://journal.uinsgd.ac.id/index.php/kp
Khazanah Pendidikan Islam, Vol. 7 No. 2: 131-148

DOI: 10.15575/kp.v7i2.46164

Bridging Misconceptions and Representations in Teaching Division: Insights from Bruner's Theory in Indonesian Classrooms

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Abstract

This qualitative study investigates elementary school teachers' conceptual understanding and instructional strategies for teaching division, with a focus on Bruner's stages of representation—enactive, iconic, and symbolic. Data were collected from 32 Indonesian teachers through open-ended prompts exploring their interpretations of division and classroom practices. Findings reveal a predominant reliance on procedural models, particularly repeated subtraction, with limited use of conceptual models like equal grouping or inverse multiplication. While many teachers employ concrete strategies, few demonstrate coherent transitions to visual and symbolic representations, resulting in fragmented instruction. The analysis also highlights inconsistencies between teachers' conceptual models and instructional methods, underscoring gaps in pedagogical content knowledge. By integrating frequency tables and case comparisons, the study identifies the need for professional development that supports representational fluency and conceptual alignment. Bruner's framework is proposed as a guide to scaffold instruction that supports diverse learners and fosters deeper mathematical understanding. Implications for teacher education and curriculum design are discussed.

Keywords: Bunner Representations, Division Operations, Elementary School Teachers, Learning Strategies, Misconceptions.

Abstrak

Studi kualitatif ini menyelidiki pemahaman konseptual dan strategi pengajaran guru sekolah dasar dalam mengajarkan operasi pembagian, dengan berfokus pada tahapan representasi menurut Bruner—enaktif, ikonik, dan simbolik. Data dikumpulkan dari 32 guru di Indonesia melalui pertanyaan terbuka yang mengeksplorasi interpretasi mereka terhadap konsep pembagian dan praktik pembelajaran di kelas. Temuan menunjukkan ketergantungan yang dominan pada model prosedural, khususnya pengurangan berulang, dengan penggunaan yang terbatas terhadap model konseptual seperti pengelompokan setara atau perkalian sebagai invers. Meskipun banyak guru menggunakan strategi konkret, hanya sedikit yang menunjukkan transisi yang koheren ke representasi visual dan simbolik, yang menyebabkan instruksi menjadi terfragmentasi. Analisis juga menyoroti adanya ketidakkonsistenan antara model konseptual guru dan metode pengajarannya, yang mengungkapkan kesenjangan dalam pengetahuan pedagogis tentang konten. Dengan mengintegrasikan tabel frekuensi dan perbandingan kasus, studi ini mengidentifikasi kebutuhan akan pengembangan profesional yang mendukung kelancaran dalam menggunakan berbagai bentuk representasi serta keselarasan konseptual. Kerangka kerja Bruner diajukan sebagai panduan untuk mendesain pembelajaran yang mendukung keberagaman peserta didik dan mendorong pemahaman matematika yang lebih mendalam. Implikasi terhadap pendidikan guru dan desain kurikulum juga dibahas.

Kata kunci: Guru Sekolah Dasar, Kesalahpahaman, Operasi Pembagian, Representasi Bruner, Strategi Instruksional,

Received: June 1, 2025; Revised: June 18, 2025; Accepted: June 19, 2025

BACKGROUND

Elementary school teachers often face challenges in effectively teaching division due to both instructional limitations and students' conceptual misunderstandings (Lamb & Booker, 2004; Unaenah et al., 2022). When division is taught with a narrow procedural focus, students may develop misconceptions that hinder their long-term mathematical performance and confidence (Nugroho et al., 2023; Purnomo et al., 2017). Many view it merely as repeated subtraction or algorithmic steps, often reflecting their teachers' procedural emphasis. Instructional practices tend to prioritize task completion over conceptual exploration, misaligning with students' cognitive development (Tarouco & Silva, 2024). Consequently, students may misinterpret division and struggle in related topics like fractions. However, research suggests that targeted professional development—emphasizing conceptual understanding and the use of mathematical representations—can improve teaching practices and foster deeper student comprehension in division and beyond.

A key factor behind the ineffective teaching of division is the limited conceptual understanding many elementary school teachers possess. While proficient in procedural techniques like long division, teachers often struggle with the underlying logic of division, including distinctions between partitive and quotitive models and connections to real-world contexts (Arrigo & Sbaragli, 2008; Simon, 1993). This leads to an overemphasis on procedural fluency at the expense of conceptual depth, resulting in rote learning and poor transfer of knowledge (Lamb & Booker, 2004). Division's complexity compared to addition or subtraction requires more nuanced teaching, which many educators are not adequately prepared for (Aytekin & Şahiner, 2020). Teachers' own misconceptions can further impede student learning, as students may become procedurally competent without true understanding—especially in areas like fractions and proportional reasoning. Research supports using strategies like problem-solving and problem-posing to help students explore part-whole relationships and various interpretations of division (Van Bommel et al., 2024). To address these issues, teacher education and professional development must emphasize conceptual foundations and representation-based pedagogy (Copur-Gencturk, 2021a; Simon, 1993). These reforms are critical, especially amid pressures from standardized curricula, to support the development of mathematically confident learners.

A key distinction in division lies between the partitive (sharing) and quotitive (grouping) models, yet this is often overlooked in practice, causing confusion and inconsistent instruction (Hopkins & Mills, 2024; Simon, 1993). Partitive division involves dividing a whole into a given number of equal parts, making it more intuitive for children (Sonja Lutovac & Maribor, 2007; Squire & Bryant, 2002). Quotitive division, on the other hand, requires determining how many groups of a certain size can be made from a whole, demanding more abstract reasoning (Eriksson et al., 2024; Kinda, 2013). Though mathematically equivalent, these models involve different cognitive processes and support distinct strategies. Many teachers default to partitive reasoning due to textbook emphasis and personal preference, which can lead to errors when applied to quotitive contexts (Lee, 2012; Weber et al., 2019). Remainders add further complexity, especially in quotitive tasks, where misrepresentation is common (Lago et al., 2008). While students perform better with partitive tasks, overreliance on this model restricts their understanding of division's broader applications, including fractions and ratios. Thus, teachers should explicitly introduce both models with contextual to develop students' flexible, conceptually grounded understanding of division.

 ISSN 2715-968X (online)

Overreliance on algorithmic instruction in division, without supporting conceptual understanding, leads to superficial learning and limits students' mathematical flexibility. Although algorithms like long division offer efficiency, teaching them in isolation often results in rote learning with little meaning (Wu, 2021). Effective instruction balances procedural fluency with conceptual insight, helping students understand both how and why procedures work (Fernandes & Martins, 2014). Many students become "prisoners of process," unable to adapt when faced with unfamiliar problems (Hurst, 2018). In contrast, combining standard algorithms with student-invented strategies—grounded in conceptual reasoning—helps bridge procedural and conceptual knowledge (Edwards & Robichaux-Davis, 2022). Teaching that emphasizes logical relationships over memorized steps fosters deeper mathematical understanding (Harris, 2023). While algorithms are important for accuracy and speed, especially in assessments, their true value emerges when taught within a conceptual framework that promotes reasoning, exploration, and transfer across mathematical contexts.

Effective use of mathematical representations is vital in teaching division, as they are not just visual aids but essential tools for helping students construct and internalize concepts. Representations—such as manipulatives, diagrams, verbal explanations, and symbols—act as cognitive bridges between abstract ideas and real-world experiences (B. Mainali, 2020; Samsuddin & Retnawati, 2018). Bruner's theory highlights a developmental sequence: enactive (hands-on), iconic (visual), and symbolic (abstract) stages, which support deep understanding (Ramsingh, 2020). However, many teachers use representations in isolation, without scaffolding across stages, which weakens students' ability to generalize concepts (Čakāne & France, 2024; Ramsingh, 2020). Limited pedagogical knowledge further hampers teachers' integration of meaningful representations, and concrete materials are often underused, especially in the enactive stage. To support conceptual flexibility, instruction should guide students through various representations—verbal, pictorial, symbolic, and numeric—in connected ways (Moon, 2024; Rahmah et al., 2019). Addressing these issues requires sustained professional development focused on improving teachers' representational competence and their ability to integrate representations into coherent, student-centered teaching (Ratumanan et al., 2022).

Manipulatives and visual aids are crucial in bridging concrete experiences and abstract reasoning in elementary mathematics. However, many teachers use these tools without explicitly linking them to core mathematical concepts, reducing them to surface-level demonstrations (Ramsingh, 2020; Gibim et al., 2023). For example, students may physically divide objects without being guided to generalize their understanding symbolically, hindering deep learning. This disconnect is particularly problematic during the transition from hands-on activities to abstract notation, which requires structured scaffolding. When used effectively, manipulatives enhance conceptual understanding by making mathematical ideas visible and interactive (Čakāne & France, 2024; Kamina & Iyer, 2009). They also support problem-solving across diverse contexts (Rosli et al., 2015). Yet, some teachers view manipulatives mainly as engagement tools, not as instruments for conceptual development (Uribe-Flórez & Wilkins, 2010). Facilitating transfer from concrete to abstract thinking demands intentional sequencing of representations and strong pedagogical insight (Cope, 2015). Therefore, professional development is essential to equip teachers with the knowledge and skills to use manipulatives meaningfully. Incorporating varied representations—visual, symbolic, verbal, and contextual—can help students form deeper connections and develop flexible, confident mathematical reasoning (Čakāne & France, 2024; Canavarro & Pinto, 2012).

One major challenge in teaching division, especially fractional division, is the gap in teachers' content knowledge and pedagogical content knowledge (PCK)—the specialized knowledge needed to make subject matter understandable to students (Shulman, 1986). PCK varies across educators and is shaped by teaching experience and classroom context (Van Driel & Berry, 2010). Teachers with limited content knowledge may rely on incomplete explanations, making it difficult for students to grasp complex ideas like part-whole relationships and flexible reference units in fractions (Copur-Gencturk, 2021). Even those with strong content knowledge may struggle to teach effectively if they lack pedagogical strategies. This is evident when representations—such as diagrams or manipulatives—are used rigidly rather than as adaptive tools for conceptual learning (Kusmaryono et al., 2019). Fractions are especially difficult due to their abstract nature, and many teachers lack a coherent approach to contextualizing them (Olanoff et al., 2014). Strengthening PCK through professional development and collaborative learning communities is essential (Olanoff, 2011; Zubaidah et al., 2023). However, improving PCK alone is not enough; curriculum, resources, and student motivation must also be addressed in a comprehensive effort to enhance mathematics teaching and learning.

Misconceptions formed during instruction can become persistent cognitive frameworks, especially when teaching emphasizes procedures over conceptual understanding (AL-Rababaha et al., 2020; Morales, 2014). For instance, students who learn that division is merely repeated subtraction may struggle with fractions or decimals, while others misapply fraction rules due to weak conceptual grounding. These misunderstandings are harder to correct when teachers lack the awareness or diagnostic skills to identify and address them (Copur-Gencturk, 2021b; Kusmaryono et al., 2019). Even experienced teachers may face difficulties, particularly in contexts like special education where clarity and scaffolding are essential. Misconceptions can also stem from students' intuitive beliefs, such as overgeneralizing whole-number properties to fractions (Ojose, 2015; Winarto et al., 2024). Diagnostic tools like structured interviews can help uncover these gaps, enabling targeted instruction (Schnepper & McCoy, 2013). Strategies such as cognitive conflict and concept re-explanation can challenge and replace incorrect beliefs with meaningful understanding (Gooding & Metz, 2009; Permata et al., 2019). High-quality professional development is essential to enhance teachers' content knowledge and pedagogical skills. Rather than viewing misconceptions as failures, educators should treat them as valuable entry points for deeper learning and instructional improvement.

The challenges in teaching division have far-reaching implications, as a weak foundation in this key operation hinders students' ability to grasp advanced concepts like proportional reasoning, algebra, and problem-solving (Boyer & Branch, 2016; Wriston, 2015). Division is vital not only for academic success but also for real-world tasks such as budgeting, measurement, and data interpretation. Yet, it is often taught merely as the inverse of multiplication, leading to superficial understanding (Hopkins & Mills, 2024). Procedural-focused instruction and insufficient curricular emphasis further contribute to student difficulties (Fauziah et al., 2023; Pujiarti & Manurung, 2024). Early mastery of division is linked to future success in mathematics and readiness for STEM careers (Wriston, 2015). Addressing these gaps requires problem-solving-based teaching that fosters number sense and emphasizes real-world contexts (Burns, 1991). Visual and hands-on approaches, along with differentiated instruction, support conceptual learning and equitable outcomes. Ultimately, strong instruction in division is essential for mathematical literacy and informed societal participation (Galbraith, 2012). This demands comprehensive reforms, including

ISSN 2715-968X (online)

curriculum improvements, targeted teacher training, and inclusive teaching materials that accommodate diverse learners and promote lifelong competence in mathematics.

This study investigates the forms and sources of misconceptions held by elementary school teachers when teaching division, focusing on the use and sequencing of representations. By analyzing qualitative teacher responses and applying frameworks like Bruner's stages of representation and Van de Walle's conceptual teaching model, the research identifies critical areas for improvement. Rather than merely critiquing current practices, the study advocates for professional development that enhances teachers' content knowledge and instructional use of representations. Addressing these issues calls for systemic changes in teacher training and support, emphasizing conceptual models, representation flexibility, and reflective pedagogy to strengthen students' mathematical understanding.

RESEARCH METHODS

This study employed a qualitative descriptive approach to examine elementary school teachers' conceptual understanding of division and their instructional strategies. The qualitative design was selected to enable an in-depth exploration of teachers' responses, particularly in relation to common misconceptions and the alignment between theoretical understanding and teaching practices.

The participants included 32 elementary school teachers from various regions in Indonesia. They were selected through purposive sampling, based on the following criteria: (1) currently teaching in grades 1 through 6, and (2) willingness to participate in a written, paper-based study that involved answering open-ended questions about the division concept. The participant pool reflected diversity in terms of geographic background, teaching experience, and exposure to professional development. All participating teachers provided informed consent before participating in the study. Their identities were anonymized, and all data were handled in accordance with ethical guidelines approved by the relevant institutional research ethics committee

Data were collected using printed questionnaires that included four open-ended questions designed to probe teachers' conceptual and pedagogical understanding of division. These questions were as follows: 1) "Provide a brief explanation of the concept of division as you understand it"; 2) "Based on your response above, what is the meaning of division according to your interpretation? Please explain your reasoning."; 3) "If you were asked to teach the concept of division to students encountering it for the first time, how would you do it?"; and 4) "In a common instructional scenario, a teacher explains 18 ÷ 6 by showing 18 marbles and distributing them equally to 6 students (A–F), who are called to the front of the classroom and each receive 3 marbles. In your opinion, does this explanation reflect the division concept you described in point 3? Please explain your response."

Thematic content analysis was used to interpret the data. Responses were read repeatedly and coded according to conceptual categories such as repeated subtraction, grouping, and the inverse of multiplication. Instructional strategies were then mapped to Bruner's representation stages: enactive (concrete manipulation), iconic (visual representation), and symbolic (abstract notation). The relationship between teachers' conceptual understanding and their proposed teaching methods was analyzed for consistency. Frequency tables were used to reveal emerging trends and associations.

To enhance the validity and reliability of the study, theoretical triangulation was applied by comparing the findings with established literature, including Bruner's theory of representation, Van de Walle's pedagogical framework, and empirical studies on mathematical misconceptions.

RESULTS AND DISCUSSION

This study explored elementary school teachers' conceptual understanding of division and the instructional strategies they employ, using data collected through a paper-based questionnaire comprising four open-ended questions. These questions prompted teachers to articulate their understanding of the concept of division, reflect on its meaning, describe how they would introduce division to first-time learners, and evaluate a sample classroom scenario in which a teacher distributed 18 marbles to 6 students. Thematic analysis of the written responses focused on two core aspects: (1) teachers' conceptual interpretations of division, and (2) the alignment between their theoretical understanding and instructional practices, particularly in explaining the division problem "18 ÷ 6." These aspects were further analyzed using Bruner's theory of representation, which emphasizes a pedagogical progression through enactive (concrete), iconic (visual), and symbolic (abstract) stages. The findings presented in this section reflect how teachers navigated these stages and how their conceptual understanding influenced their teaching decisions.

Teachers' Conceptual Understanding of Division

The majority of teachers (24 out of 32) described division using the repeated subtraction model, such as solving 18 ÷ 6 by computing 18 − 6 − 6 − 6 = 0. Only 5 teachers explained division through equal grouping, while 3 mentioned it as the inverse of multiplication. These patterns suggest that many educators approach division procedurally rather than conceptually. This distribution is summarized in Table 1.

Table 1. Distribution of Teachers' Conceptual Models of Division

Conceptual Understanding	Number of Teachers	Percentage
Repeated Subtraction	24	75%
Equal Grouping	5	15.6%
Inverse of Multiplication	3	9.4%

The findings from the provided research papers underscore the dominance of procedural teaching in division instruction, highlighting a significant gap in the integration of relational and conceptual frameworks. Procedural knowledge often overshadows conceptual understanding, as traditional teaching methods tend to emphasize algorithmic reasoning, such as repeated subtraction, rather than diverse conceptual models (Arslan, 2010; Simon, 1993). Although repeated subtraction is a valid strategy, its overuse limits students' capacity to understand division in more complex contexts involving fractions, ratios, or remainders (Adu-Gyamfi et al., 2019; Weber et al., 2019) This reliance on procedural techniques reflects a broader issue in mathematics instruction—teachers' limited exposure to and confidence with developmentally appropriate models like equal grouping (Downton, 2008). In addition, many educators struggle to represent the inverse relationship between multiplication and division in a meaningful, visual, or contextual manner (Robinson & LeFevre, 2012; Timmerman, 2014).

To address these challenges, research calls for comprehensive professional development focused on deepening teachers' understanding of partitive and quotitive division, and on

 ISSN 2715-968X (online)

strengthening their ability to apply these frameworks in varied instructional settings(Aytekin & Şahiner, 2020; Timmerman, 2014). Teachers frequently exhibit weak conceptual knowledge in linking symbolic representations of division to real-world applications and in identifying reference units during computations (Simon, 1993). Additionally, cognitive obstacles—such as confusion between partitive and quotitive models and difficulties with re-unitization—further hinder teachers' ability to model division conceptually (Weber et al., 2019). The slow development of understanding the inverse relationship between multiplication and division compounds these difficulties and highlights the need for instruction that fosters flexible problem-solving skills (Robinson & LeFevre, 2012). While shifting toward a more conceptually grounded approach is essential, this transition poses significant challenges and underscores the importance of supporting teachers with ongoing training that integrates both procedural fluency and conceptual depth to promote meaningful student learning in division.

Evaluation of the Teaching Scenario (18 ÷ 6)

When asked to evaluate a sample teaching scenario involving distributing 18 marbles to 6 children, 22 teachers (68.8%) agreed that the method aligned with their conceptual model of division. However, 10 teachers (31.2%) expressed concern about the lack of explicit explanation and limited student engagement. This response suggests an awareness of the importance of active learning and guided exploration, essential features of Bruner's enactive stage.

Table 2. Teachers' Evaluation of a Division Teaching Scenario

Evaluation of Teaching Scenario	Number of Teachers	Percentage
Appropriate	22	68.8%
Inappropriate	10	31.2%

The analysis of teachers' instructional responses reveals a predominant reliance on enactive strategies, where concrete materials such as marbles, candy, or stones are used to introduce the concept of division. This approach aligns well with Bruner's enactive stage and is developmentally appropriate for early learners. However, only a few teachers demonstrated a coherent instructional sequence that progresses through Bruner's full representation model. For example, Teacher 1 and Teacher 11 exemplified best practices by integrating enactive, iconic, and symbolic representations—starting with hands-on activities, transitioning to pictorial diagrams, and concluding with symbolic notation (e.g., 18 ÷ 6 = 3). This structured transition is essential for scaffolding students' understanding and fostering deeper mathematical reasoning.

In contrast, several teachers, including Teacher 3 and Teacher 10, stopped at the enactive stage, focusing solely on physical manipulation without connecting it to visual or symbolic forms. Others, like Teacher 12 and Teacher 15, made partial attempts to bridge to the iconic stage by using drawings, but still fell short of introducing formal mathematical symbols. This pattern indicates a gap in instructional depth, where the absence of visual and symbolic scaffolding may hinder students from internalizing abstract concepts. Thus, while enactive strategies are widely implemented, the lack of progression to higher representation stages highlights the need for professional development focused on comprehensive representational teaching strategies.

Table 3. Analysis of Teachers' Use of Representational Stages in Division Instruction

Teacher Code	Enactive (Concrete Use)	Iconic (Images/Drawings)	Symbolic (Mathematical Notation)	Representation Transition Description
1	✓	✓	✓	Full transition from concrete to symbolic representations
3	✓	✗	✗	Stopped at concrete stage without further representations
10	✓	✗	✗	Manipulative use only, no symbolic or iconic reference
11	✓	✓	✓	Complete and coherent use of representations
12	✓	✓	✗	Partial transition, no symbolic use
15	✓	✓	✗	Partial use of visual representation, no mathematical symbols
17	✓	✗	✗	Only enactive strategies used

The question highlights a critical pedagogical issue: the varying ability of teachers to effectively use representations as tools for fostering conceptual understanding in mathematics, rather than as mere illustrations. This discrepancy is evident in how teachers approach division, with some viewing it as a straightforward demonstration and others emphasizing the value of guided discovery. Teachers often exhibit partial consistency between their conceptual models and instructional strategies. For instance, those who understand division as repeated subtraction tend to favor symbolic or sequential approaches, while those who view it as equal grouping typically employ hands-on, concrete methods. However, inconsistencies frequently arise when teachers' conceptual understanding does not fully translate into aligned pedagogical practices (Ramsingh, 2020; Timmerman, 2014). Ramsingh's findings further show that teachers often fail to employ representations meaningfully, exposing gaps in both content knowledge and pedagogical application.

Representations are vital for helping students build connections between mathematical ideas and progress from concrete experiences to abstract reasoning (Čakāne & France, 2024). When used effectively, tools such as manipulatives, diagrams, and symbols support students in grasping complex topics like division and fractions (Jao, 2013)). However, despite their recognized importance, many teachers rely on a single form of representation, limiting students' opportunities for deeper understanding (Sa'dijah et al., 2018). To bridge this gap, professional development programs must equip teachers with the skills to design lessons that integrate multiple representations effectively. These programs should enhance both content knowledge and pedagogical strategies, emphasizing the cognitive transition from enactive to iconic and symbolic reasoning as proposed in Bruner's theory (Niebert et al., 2013). Furthermore, Timmerman (2014) highlights the need for training that includes contextual problem design and clarity in representing division outcomes, especially those involving remainders. Ultimately, while the integration of representations is widely accepted as beneficial, sustained support through professional learning is necessary to ensure teachers can apply these strategies consistently and meaningfully in the classroom.

Alignment Between Understanding and Instructional Strategy

The analysis revealed partial consistency between teachers' conceptual models and their instructional approaches. Teachers who conceptualized division as repeated subtraction typically employed symbolic or sequential methods, such as step-by-step subtraction, which showed consistent alignment between understanding and practice. Similarly, those who understood division as equal grouping used concrete distribution strategies, such as physically grouping objects, which also demonstrated consistency.

In contrast, teachers who perceived division as the inverse of multiplication often adopted mixed or abstract approaches. This group exhibited less consistent alignment, possibly due to the abstract nature of the concept and the challenges in translating this understanding into concrete instructional practices. These findings suggest that while procedural models are commonly and coherently applied, more abstract conceptual models require greater pedagogical support to ensure consistent and effective instruction.

Tabel 4. Alignment Between Conceptual Models and Instructional Strategies in Division Teaching

Conceptual Model	Strategy Used	Consistency
Repeated Subtraction	Symbolic/Sequential	Consistent
Equal Grouping	Concrete Distribution	Consistent
Inverse of Multiplication	Mixed/Abstract	Less clear

Teachers' conceptual models significantly shape the instructional strategies they adopt in the classroom. Those who view division as repeated subtraction often rely on symbolic or sequential methods, focusing on deductive steps where a number is repeatedly reduced by the divisor until a remainder is reached (Arrigo & Sbaragli, 2008; Ramsingh, 2020). This approach reflects procedural fluency but can become limiting if not connected to broader mathematical reasoning. On the other hand, teachers who conceptualize division as equal grouping frequently employ hands-on techniques, such as using physical objects to form groups. This method is often more accessible to young learners and aligns with Bruner's enactive stage, offering a concrete foundation for abstract understanding(Adu-Gyamfi et al., 2019; Ramsingh, 2020). Despite these aligned practices, inconsistencies are observed in classrooms where teachers with strong conceptual understanding struggle to implement corresponding instructional strategies. These mismatches suggest barriers in pedagogical content knowledge—the ability to translate abstract mathematical ideas into effective teaching methods (Arrigo & Sbaragli, 2008; Ramsingh, 2020).

A key factor contributing to these inconsistencies is the ineffective use of representations. Teachers may use drawings or manipulatives, but often treat them as illustrative endpoints rather than tools for inquiry and reasoning, thereby limiting their pedagogical potential (Ramsingh, 2020). In many cases, teachers possess sound conceptual knowledge yet lack the instructional expertise to scaffold this understanding effectively (Lee, 2012). To address this gap, professional development should focus on integrating cognitive models into lesson planning and delivery. These models provide structured approaches for instructional design that can help teachers bridge theory and practice, improving the coherence between what they know and how they teach (English & Halford, 2012; Rittle-Johnson & Koedinger, 2001);. Ultimately, strengthening this alignment is essential not only for enhancing teacher efficacy but also for fostering meaningful student learning experiences.

Theoretical Integration: Bridging Representations Through Bruner's Framework

Jerome Bruner's theory of representation—enactive, iconic, and symbolic stages—provides a vital framework for developing mathematical understanding in elementary students, aligning with Piaget's concrete operational stage (Bruner & Kenney, 1965; Zuliana et al., 2019). Yet, many teachers emphasize symbolic methods while overlooking the enactive and iconic stages critical for conceptual learning (Čakāne & France, 2024; Samsuddin & Retnawati, 2018) This gap stems from limited training, resource constraints, and rigid curricula (Lesser & Tchoshanov, 2005) . When used effectively, Bruner's stages can raise conceptual mastery significantly and enhance problem-solving (Gombo, 2024; Rahmah et al., 2019; Wibowo et al., 2023).

Manipulatives, central to the enactive stage, help young learners build foundational understanding through hands-on engagement (D'Angelo & Iliev, 2012; Yusof et al., 2013). However, when not connected to visual or symbolic models, their impact diminishes, leading to superficial learning (Kamina & Iyer, 2009). Effective integration requires structured reflection and professional development (Larkin, 2016; Mcdonough, 2016).

The underutilization of visual representations in elementary mathematics education is a key barrier to learning, as they bridge concrete experiences and abstract reasoning. Tools like diagrams and pictorial models help generalize concepts and improve problem-solving by making abstract ideas more accessible (Booth & Koedinger, 2010; Čakāne & France, 2024; Johnny, 2024). However, their use is often limited to static visuals with little interaction or conceptual depth, reducing effectiveness (Kamina & Iyer, 2009). This reflects gaps in teacher training and integration strategies (Johnny, 2024) When used meaningfully, visual tools improve understanding, engagement, and student attitudes toward mathematics (Parame-Decin, 2023).

Similarly, introducing symbolic representations prematurely—without enactive or iconic groundwork—often leads to rote learning and weak conceptual grasp (Maffia & Mariotti, 2020). Students need concrete experiences and visual reasoning to understand abstract symbols (Draper, 2003; Rubenstein & Thompson, 2001)). Teachers must scaffold this progression and use differentiated strategies to support diverse learning needs (Downton, 2008).

The overemphasis on symbolic proficiency in mathematics education often weakens conceptual understanding, resulting in fragile knowledge that limits flexible application. In division, students need strong mental representations and adaptable thinking—skills not developed through symbols alone (Heeffer, 2013; Jupri & Sispiyati, 2020). Symbols, while compact, can hinder learning if introduced without grounding in concrete or visual experiences. Many students show procedural skill but lack "symbol sense," complicating topics like algebra and fractions (Tall et al., 2001). Concrete approaches that link materials, language, and symbols enhance understanding (Coles & Sinclair, 2019; Norton & Irvin, 2007). Teachers must bridge symbolic and conceptual knowledge to ensure meaningful learning (Rubenstein & Thompson, 2001).

Bruner's stages are often poorly implemented due to gaps in teacher training. Many educators receive minimal exposure to cognitive theory and default to procedural curricula (Nur Arsyad et al., 2024; Popova & Jones, 2021; Zuliana et al., 2019). To address this, teacher education should embed cognitive theory, develop representational skills, and support transitions across stages (Barth, 2015; Gilbert & Eilam, 2014; Soloman, 2024).

To address persistent gaps in professional development, it is essential to center training on Bruner's representation theory, which emphasizes the cognitive progression from enactive to iconic and symbolic stages (Leão & Goi, 2021; Lopes, 2024). This framework supports both student learning and instructional design. Teachers need structured opportunities to develop lesson sequences that integrate all three stages (Orlofsky, 2001), using collaborative design, peer observation, and reflective video analysis (Andersen et al., 2009; Hubber, 2010; López & Larrea, 2017). Successful application also depends on managing cognitive load and adapting strategies for diverse contexts (Tytler et al., 2017).

Equipping teachers with strategies to bridge these stages enhances students' understanding. Activities like acting out problems, sketching visuals, and writing symbolic equations help build logical reasoning. Guiding questions such as "What do the objects show?" promote engagement and reflection. Visuals link concrete to abstract thinking and improve communication (Burtness, 2003; Monoyiou et al., 2007). Visualization aids metacognition (Du Plooy, 2015), while strategies like problem-based learning and drawing foster independent, holistic learning (Cobeña & Loor, 2023; Dirkes, 1991).

Student engagement is essential for effective learning, especially through Bruner's stages—enactive, iconic, and symbolic. Each requires distinct participation: physical manipulation in the enactive stage fosters procedural understanding (Ross & Willson, 2012), student-generated visuals in the iconic stage reveal misconceptions and support concept formation (Kenny & Cirkony, 2017), and articulation in the symbolic stage promotes critical thinking (ElSayary, 2023). Engagement is multifaceted—cognitive, emotional, and behavioral—and must be sustained with clear expectations and responsive teaching (Parsons et al., 2014; Powell & Stringer, 2018).

Van de Walle emphasizes teaching division in meaningful contexts, moving beyond rote learning toward real-world applications like equal sharing and grouping (Dopico Crespo, 2001). This problem-solving focus enhances reasoning (Burns, 1991) and is supported by representations—visual, manipulative, and symbolic (Milton et al., 2019). Methods like group discussions and CRA sequencing scaffold learning across representational stages (Maralova, 2024). Integrating division with multiplication strengthens fluency and relational understanding (Young-Loveridge & Bicknell, 2018). However, effective implementation requires targeted teacher training and curriculum support to ensure transitions between representations are coherent and inclusive.

Implementing Bruner's enactive-iconic-symbolic sequence supports equity in math education by reducing cognitive load and enhancing understanding for students with diverse needs, including those with language barriers and learning disabilities. Multimodal representations—verbal, visual, numeric, and algebraic—allow flexible thinking and aid knowledge transfer (Banerjee & Gautam, 2024; Mainali, 2020). Concrete tools and virtual manipulatives promote engagement and accessibility (D'Angelo & Iliev, 2012; Siller & Ahmad, 2024). Visual aids also help students communicate mathematically beyond language constraints (Monoyiou et al., 2007) but realizing these benefits requires thoughtful resource design and systemic teacher training (Ajayi & Lawani, 2015).

This study emphasizes the need for an integrated approach to representation in division instruction. While many teachers use manipulatives, few extend this to structured visual and symbolic scaffolding (Ramsingh, 2020). Bridging stages is essential for deep understanding, yet teacher preparation often neglects representational sequencing (Timmerman, 2014). Professional development should focus on designing contextual problems, reflective strategies, and cross-stage integration (Gibim et al., 2023; Stohlmann et al., 2019), empowering students to connect and apply mathematical concepts flexibly across contexts.

This study offers valuable insights into elementary school teachers' conceptual understanding and instructional strategies for teaching division. However, several limitations must be acknowledged. First, the small sample size—32 purposively selected teachers—limits the generalizability of the findings to broader populations. Second, relying solely on printed open-ended questionnaires as the data collection method may have constrained the depth of participants' responses, especially in the absence of follow-up interviews or classroom observations that could provide richer contextual information. Third, the use of self-reported data introduces the possibility of social desirability bias, where teachers may describe idealized practices rather than their actual classroom behavior. Fourth, the study does not directly link teachers' conceptual understanding with student learning outcomes, making it difficult to assess the practical impact of the instructional strategies discussed. Fifth, the descriptive and cross-sectional nature of the study limits the ability to track changes in teachers' practices or understanding over time.

CONCLUSION

This study reveals that misconceptions about division remain prevalent among elementary teachers, particularly the procedural framing of division as repeated subtraction. While concrete strategies are frequently used, transitions to iconic and symbolic representations are often absent or poorly structured, limiting students' conceptual understanding. Moreover, teachers who demonstrate a conceptual grasp of division do not always apply aligned instructional strategies, indicating a gap in pedagogical content knowledge. To address this, professional development grounded in Bruner's representational sequence—from enactive to iconic to symbolic—is essential to support effective, conceptually rich instruction. Therefore, further research is recommended to investigate the impact of targeted training programs on teachers' representational fluency and classroom practices, ideally through longitudinal studies. Future studies should also examine how representation-based strategies influence student learning outcomes, especially in complex topics such as fractions and proportional reasoning, and how such approaches can be adapted across diverse educational settings. Exploring the role of reflective teaching and the integration of digital tools may also provide valuable insights for enhancing mathematics education.

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